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Upper and Lower Limits on Neutralino WIMP Mass and Spin–Independent Scattering Cross Section, and Impact of New $(g-2)_\mu$ Measurement

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Abstract: We derive the allowed ranges of the spin–independent interaction cross section σ_p^{SI} for the elastic scattering of neutralinos on proton in the general Minimal Supersymmetric Standard Model. We investigate the effects of the lower limits on Higgs and superpartner masses from colliders, as well as the impact of constraints from $b \to s\gamma$ and the new measurement of $(g-2)_\mu$. We further explore the effect of the neutralino relic density, including coannihilation, and of theoretical assumptions about the largest allowed values of supersymmetric parameters on deriving the lower limits on the scattering cross section. For $\mu > 0$, requiring the latter to lie below 1 TeV leads to restricting $\sigma_p^{SI} \gtrsim 10^{-11}$ pb at $m_\chi \sim 100$ GeV up to $\sigma_p^{SI} \gtrsim 10^{-8}$ pb at $m_\chi \sim 1$ TeV. When supersymmetric parameters are allowed above 1 TeV, for $m_\chi \gtrsim 440$ GeV we derive a *parameter–independent lower limit* of $\sigma_p^{SI} \gtrsim 2\times 10^{-12}$ pb. (No similar lower limits can be set for $\mu < 0$.) The new measurement of $(g-2)_\mu$ restricts $m_\chi \lesssim 420$ GeV at 1σ CL and $m_\chi \lesssim 800$ GeV at 2σ CL, and implies $\mu > 0$. The largest allowed values of σ_p^{SI} have already become accessible to recent experimental searches.

Keywords: Supersymmetric Effective Theories, Cosmology of Theories beyond the SM, Dark Matter.

Contents

1. Introduction

The hypothesis of the lightest neutralino χ, as the lightest supersymmetric particle (LSP), providing the dominant contribution to cold dark matter (CDM) in the Universe, has inspired much activity in the overlap of today's particle physics and cosmology. It is well–known that the relic density of the neutralinos is often comparable with the critical density [1, 2]. Likewise, the expectation that the Galactic dark matter (DM) halo is mostly made of weakly–interacting massive particles (WIMPs) has further led to much experimental activity. In particular, the experiments looking for the elastic scattering of CDM WIMPs off underground targets have recently set limits on spin–independent (SI), or scalar, cross section of the order of 10^{-6} pb [3, 4, 5]. They have also nearly ruled out the region of (m_χ, σ_p^{SI}) that has been claimed by the DAMA experiment to be consistent with an annual modulation effect [6]. Initial and early studies [7, 8, 9, 10] were followed by more recent work [11], where it was concluded that such levels are generally comparable with the ranges expected from the neutralino WIMP in the Minimal Supersymmetric Standard Model (MSSM). They are, however, still at least on order of magnitude, or so, above the ranges predicted by recent analyses of the Constrained MSSM (CMSSM) [12, 13, 14, 15, 16].

For comparison, cross sections for spin–dependent (SD) interactions are in the case of the neutralino generally some two or three orders of magnitude larger than the SI ones. On the other hand, at present detectors are still not sensitive enough to explore the parameter space of the MSSM, despite recent progress [17].

In light of the ongoing and planned experimental activities, it is timely to conduct a thorough and careful re–analysis of the predicted cross sections for SI scattering of neutralino WIMPs. Such a study is rather challenging because resulting ranges often strongly depend on a given SUSY model and on related theoretical assumptions. They are further affected by experimental limits on SUSY, both from colliders and from indirect searches, as well as by cosmological input, where the relic abundance of the CDM has been measured with better accuracy both directly and in CMBR studies [18]. Over the last few years and months there have been also new results for LEP lower bounds on the masses of the lightest Higgs and electroweakly–interacting superpartners, Tevatron lower limits on strongly–interacting superpartners, as well as limits on allowed SUSY contributions to $b \to s\gamma$, and especially to the anomalous magnetic moment of the muon $(g-2)_\mu$ [19]. The new result for $(g-2)_\mu$ indicates a sizable deviation from the Standard Model prediction, whose value is still a subject of much discussion. As we will show, when interpreted in terms of SUSY, the required extra contribution to $(g-2)_\mu$ plays a unique role in implying a stringent *upper* bound $m_\chi \lesssim 420\,\mathrm{GeV}$ (1σ CL) and $m_\chi \lesssim 800\,\mathrm{GeV}$ (2σ), but it does *not* affect much the allowed ranges of the SI scattering cross section σ_p^{SI}.

In this Letter we carefully study the impact of the above constraints. In an attempt to minimize theoretical bias, we work here in the context of the general MSSM, which will be defined below, with an additional assumption of R–parity conservation. We focus here on the SI cross section case. Other recent studies of the general MSSM include [20, 21, 22]. The results presented here show that the level of experimental sensitivity that has recently been reached [3, 4, 5] has now indeed allowed one to start exploring cosmologically favored ranges of the neutralino WIMP mass and SI cross section. However, we point out a number of caveats and relations and further discuss the origin of, and robustness of, the upper and lower limits on σ_p^{SI}. In particular, at large enough m_χ we are able to derive *parameter–independent* lower bounds on σ_p^{SI}.

2. The MSSM

We start by reviewing relevant features of the general MSSM [23]. (We follow the convention of Ref. [24].) By the MSSM we mean a supersymmetrized version of the Standard Model, with Yukawa and soft SUSY–breaking terms consistent with R–parity. We neglect CP–violating phases in the Higgs and SUSY sectors and assume no mixings among different generations of squarks and sleptons, since both are probably small. In the MSSM, all the physical slepton and squark masses can be considered as a priori free parameters set at the electroweak scale. In the same way we treat the trilinear parameters A_i ($i = t, b, \tau$) of the third generation while neglecting the ones of the first two. The Higgs sector is determined at the tree level by the usual ratio of the neutral Higgs VEV's $\tan\beta = v_t/v_b$ and the mass of the pseudoscalar m_A. In computing full one-loop and leading two-loop radiative corrections to the lightest scalar Higgs we use the package FeynHiggsFast (FHF) [25]. As we will see, Higgs masses will play an important role in the analysis.

The lightest neutralino χ is lightest of the four mass eigenstates of the linear combinations of the bino $\widetilde{B}$, $\widetilde{W}_3^0$ and the two higgsinos $\widetilde{H}_b^0$ and $\widetilde{H}_t^0$

$$\chi \equiv \chi_1^0 = N_{11}\widetilde{B} + N_{12}\widetilde{W}_3^0 + N_{13}\widetilde{H}_b^0 + N_{14}\widetilde{H}_t^0. \tag{2.1}$$

The neutralino mass matrix $\mathcal{M}$ [24] is determined by the $U(1)_Y$ and $SU(2)_L$ gaugino mass parameters M_1 and M_2, respectively (and we impose the relation $M_1 = \frac{5}{3}\tan^2\theta_W M_2$, which comes from assuming gaugino mass unification at GUT scale), the Higgs/higgsino mass parameter μ, as well as $\tan\beta$. In the region $|\mu| \gg M_1$, the lightest neutralino is mostly a bino with mass $m_\chi \simeq M_1$. In the other extreme, it is mostly higgsino–dominated and $m_\chi \simeq |\mu|$.

For the purpose of this analysis, we take as independent parameters: $\tan\beta$, μ, M_2, m_A, $A_{t,b}$ as well as the masses of the sleptons and of the squarks. In order to make our analysis manageable, we make an additional assumption that, at the electroweak scale, the soft mass parameters of the sleptons are all equal to some common value $\widetilde{m}_{\widetilde{l}}$, and analogously $\widetilde{m}_{\widetilde{q}}$ for all the squarks. One normally expects certain relations among the physical masses of the sleptons and squarks since they receive in addition well–defined

D–term and F–term contributions to their mass matrices. Assuming common soft mass terms, at either GUT or electroweak scale, this normally leads to the sleptons being lighter than the squarks. Furthermore, for the sfermions of the 3rd generation it is natural to expect large mass splittings. However, we believe that, for our purpose, a simplification of introducing just two separate common soft mass scales $\widetilde{m}_{\widetilde{l}}$ and $\widetilde{m}_{\widetilde{q}}$, while greatly simplifying the analysis, will not play much role in our overall conclusions for the SI cross sections. (This is in contrast to often assumed *full* degeneracy of soft sfermion masses which leads in our opinion to an unnecessary limitation on the allowed SUSY parameter space.)

What we do find important is to disentangle squark and slepton masses. This is mainly because experimental limits on slepton masses are significantly weaker than in the case of squarks. For the case of bino–like neutralino, which is the most natural case for providing $\Omega_\chi h^2 \sim 1$ [26], it is therefore the mass of the lightest slepton which often predominantly determines $\Omega_\chi h^2$. Assuming common soft masses for sleptons and squarks at the electroweak scale would therefore generally lead to overestimating $\Omega_\chi h^2$ by missing the cases where relatively light sleptons (below current squark mass bounds) would otherwise provide acceptable $\Omega_\chi h^2$. An additional effect is that of coannihilation. When slepton mass is only somewhat larger than that of the LSP, the neutralino relic abundance is strongly reduced and otherwise forbidden cases become allowed [27].

3. SI Cross Section

For non-relativistic Majorana particles, like the neutralino WIMP, the elastic scattering off constituent quarks and gluons of some nucleon ${}^A_Z X$ is given by an effective differential cross section [7, 8, 10]

$$\frac{d\sigma}{d|\vec{q}|^2} = \frac{d\sigma^{SI}}{d|\vec{q}|^2} + \frac{d\sigma^{SD}}{d|\vec{q}|^2}, \tag{3.1}$$

where the transferred momentum $\vec{q} = \mu_A \vec{v}$ depends on the velocity $\vec{v}$ of the incident WIMP, and $\mu_A = m_A m_\chi/(m_A + m_\chi)$ is the reduced mass of the system. The effective WIMP-nucleon cross sections σ^{SI} and σ^{SD} are computed by evaluating nucleonic matrix elements of corresponding WIMP–quark and WIMP–gluon interaction operators.

In the SI part, contributions from individual nucleons in the nucleus add coherently and the finite size effects are accounted for by including the SI nuclear form factor $F(q)$. The differential cross section for the scalar part then takes the form [2]

$$\frac{d\sigma^{SI}}{d|\vec{q}|^2} = \frac{1}{\pi v^2}\left[Z f_p + (A-Z) f_n\right]^2 F^2(q), \tag{3.2}$$

where f_p and f_n are the effective neutralino couplings to protons and neutrons, respectively. Explicit expressions for the case of the supersymmetric neutralino can be found, *e.g.*, in [28]. The formalism we follow has been reviewed in several recent papers [2, 28, 12]. We have re–done the original complete calculation of Drees and Nojiri [10] and agreed with their results.

A convenient quantity which is customarily used in comparing theory and experimental results for SI interactions is the cross section σ_p^{SI} for WIMP elastic scattering of free proton in the limit of zero momentum transfer:

$$\sigma_p^{SI} = \frac{4}{\pi}\mu_p^2 f_p^2 \qquad (3.3)$$

where μ_p is defined similarly to μ_A above. The analogous quantity for a target with nuclei with mass number A can then be expressed in terms of σ_p^{SI} as

$$\sigma_A^{SI} = \frac{4}{\pi}\mu_A^2 \left[Z f_p + (A-Z) f_n\right]^2 = \left(\frac{\mu_A}{\mu_p}\right)^2 A^2 \sigma_p^{SI}. \qquad (3.4)$$

One can do so because, for Majorana WIMPs, $f_p \simeq f_n$.

The coefficients $f_{p,n}$ can be expressed as [10]

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} \frac{f_{Tq}^{(p)}}{m_q} f_q \ + \ \frac{2}{27} f_{TG}^{(p)} \sum_{c,b,t} \frac{f_q}{m_q} + ...$$

where $f_{TG}^{(p)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(p)}$, and $f_{Tq}^{(p)}$ is given by $< p|m_q \bar{q} q|p >= m_p f_{Tq}^{(p)}$ $(q = u, d, s)$, and analogously for the neutron. The masses and ratios $B_q = \langle p|\bar{q}q|p\rangle$ of light constituent quarks in a nucleon come with some uncertainties. For definiteness, we follow a recent re-evaluation [12] and assume $m_u/m_d = 0.553 \pm 0.043$, $m_s/m_d = 18.9 \pm 0.8$, and $B_d/B_u = 0.73 \pm 0.02$, as well as

$$f_{Tu}^{(p)} = 0.020 \pm 0.004,\ \ f_{Td}^{(p)} = 0.026 \pm 0.005,\ \ f_{Ts}^{(p)} = 0.118 \pm 0.062$$

$$f_{Tu}^{(n)} = 0.014 \pm 0.003,\ \ f_{Td}^{(n)} = 0.036 \pm 0.008,\ \ f_{Ts}^{(n)} = 0.118 \pm 0.062.$$

which numerically gives SI cross section values very similar to using the set of [2]. Some other recent studies use a new determination of $\sigma_{\pi N}$ to derive a much larger value for $f_{Ts}^{(p)} \simeq 0.37$ [29]. We find such values somewhat questionable since they imply that the strange quark component of the nucleon would be larger than the up and down ones. We have numerically checked that using the set of input parameters of [29] gives typically SI cross section values a factor of six higher than in our case.

It is worth noting that, despite several different diagrams and complicated expressions, it is the exchange of the heavy scalar Higgs that in most cases comes out to be numerically dominant. It is further enhanced when the neutralino is a mixed gaugino–higgsino state [30]. As $m_H \simeq m_A$ increases, σ_p^{SI} drops as m_H^{-4} because of the t–channel propagator effect in $\chi q \rightarrow \chi q$ elastic scattering. Eventually, at smaller σ_p^{SI}, squark exchange becomes important instead.

4. Details of the Scan

As outlined above, we use seven parameters $\tan\beta$, M_2, μ, m_A, $\widetilde{m}_{\widetilde{q}}$, $\widetilde{m}_{\widetilde{l}}$ and $A_t = A_b$ to conduct a careful scan of the general MSSM parameter space. For their allowed ranges we take:

$$
\begin{aligned}
50\,\mathrm{GeV} \leq \ & M_2 \ \leq 2\,\mathrm{TeV} \\
50\,\mathrm{GeV} \leq \ & |\mu| \ \leq 2\,\mathrm{TeV}\ (4\,\mathrm{TeV}) \\
50\,\mathrm{GeV} \leq \ & \widetilde{m}_{\widetilde{l}} \ \leq 2\,\mathrm{TeV}\ (4\,\mathrm{TeV}) \\
200\,\mathrm{GeV} \leq \ & \widetilde{m}_{\widetilde{q}} \ \leq 2\,\mathrm{TeV}\ (4\,\mathrm{TeV}) \\
90\,\mathrm{GeV} \leq \ & m_A \ \leq 2\,\mathrm{TeV} \\
0 \leq \ & |A_{t,b}| \leq 1\,\mathrm{TeV} \\
5 \leq \ & \tan\beta \leq 50
\end{aligned}
$$

while we set $A_\tau = 0$ since we treat slepton masses as independent parameters anyway. In addition to a general scan of the parameter space, in many areas we do several focused scans and explore the effect of extremely large values of μ, $\widetilde{m}_{\widetilde{l}}$ and $\widetilde{m}_{\widetilde{q}}$ beyond $2\,\mathrm{TeV}$ (given in brackets above) in order to derive *parameter–independent* lower limits on σ_p^{SI}, as described below. The minimum values are set so that the resulting physical masses of Higgs and superpartners are limited from below by collider bounds. For the lighter chargino we take $m_{\chi_1^\pm} > 104\,\mathrm{GeV}$ [31], for sleptons the lower limit of $90\,\mathrm{GeV}$ [31] and for squarks $200\,\mathrm{GeV}$ [32]. The lower limit on m_χ depends not only on a model but also on a number of additional assumptions [33]. For this reason, in our analysis we conservatively do not impose a direct experimental limit on m_χ, but instead infer it from the other limits, especially the one on the chargino mass. As regards the lightest Higgs mass m_h, in much of the parameter space ($m_A > 120\,\mathrm{GeV}$) the lower limit on the Standard Model Higgs of $114.1\,\mathrm{GeV}$ [34] applies. However there are two important points to note. Firstly, theoretical uncertainties in computing m_h in the MSSM are estimated at $2-3\,\mathrm{GeV}$. Conservatively, we thus require only $m_h > 111\,\mathrm{GeV}$. Secondly, for $90\,\mathrm{GeV} < m_A < 120\,\mathrm{GeV}$, there still remains a sizable range of the (m_h, m_A)–plane where the lightest Higgs mass given roughly by $m_h > 0.78(m_A + 21.7\,\mathrm{GeV})$ is allowed [34]. We will comment below on the sizable effect of this low–mass range on increasing the largest allowed values of σ_p^{SI}.

Among indirect limits on SUSY, $b \to s\gamma$ often places an important additional constraint on the allowed parameter space. We calculate the SM contribution to $\mathrm{BR}(B \to X_s\gamma)$ at the full NLO level and include dominant $\tan\beta$–enhanced NLO SUSY [35], and also include the c-quark mass effect on the SM value [36]. As described in detail in [14], with an update in [37], we allow full SM+SUSY contributions to be in the range $\mathrm{BR}(B \to X_s\gamma) = (3.23 \pm 0.72) \times 10^{-4}$. It is important, however, to stress here an important salient point. In computing SUSY contribution to $\mathrm{BR}(B \to X_s\gamma)$ one usually makes an implicit assumption of minimal flavor violation in the down–type squark sector, which is theoretically poorly justified. Even a slight modification of the assumption often leads to a significant relaxation of the bound from $b \to s\gamma$ to the point of even allowing $\mu < 0$ and relatively light superpartner masses [38].

A very recent measurement of the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ [19] has confirmed a previous value [39] but with twice–increased precision [19]

$$
a_\mu^{\mathrm{expt}} - 11659000 \times 10^{-10} = (204 \pm 8) \times 10^{-10}. \tag{4.1}
$$

Further clarification is required, and soon expected, on the theory side, regarding especially the size and errors in the hadronic vacuum polarization and light–by–light corrections. At this point, we assume the value $a_\mu^{\rm SM} - 11659000 \times 10^{-10} = (176 \pm 7) \times 10^{-10}$ obtained in [40], with corrections for light–by–light from [41]. This leads to a 2.6σ discrepancy

$$\Delta a_\mu = a_\mu^{\rm expt} - a_\mu^{\rm SM} = (28 \pm 11) \times 10^{-10}, \tag{4.2}$$

which restricts possible SUSY contribution to

$$17 \times 10^{-10} < a_\mu^{\rm SUSY} < 39 \times 10^{-10} \quad (1\sigma) \tag{4.3}$$

$$6 \times 10^{-10} < a_\mu^{\rm SUSY} < 50 \times 10^{-10} \quad (2\sigma), \tag{4.4}$$

and also selects $\mu > 0$ (the sign of the SUSY contribution is the same as that of μ). Since some other recent evaluations give a larger value of a_μ^{SM} [42] and larger error bar [43] but also the very recent ones [44] seem to obtain a_μ^{SM} in agreement with [40], at this point, we will not strictly impose the $(g-2)_\mu$ constraint on the whole otherwise allowed parameter space. However, we will display the important impact of $(g-2)_\mu$ on m_χ of applying the constraints (4.4) and (4.4) on otherwise allowed ranges of parameters.

For the relic abundance, a lower limit on the age of the Universe conservatively gives $\Omega_\chi h^2 < 0.3$, while "direct" measurements of the CDM lead to $0.1 < \Omega_\chi h^2 < 0.2$ which we will treat as a preferred range. Recent precision studies of the CMBR imply more narrow ranges, like $0.10 < \Omega_\chi h^2 < 0.12$. We will not apply it here because it does not appear to have much impact on the upper and/or lower limits on σ_p^{SI}, although it does exclude a number of otherwise allowed points. We compute $\Omega_\chi h^2$ as accurately as one reasonably can, both near and further away from poles and thresholds by applying our recently derived exact analytic expressions for neutralino pair–annihilation [45] and neutralino–slepton coannihilation [46], and by using exact procedure for the neutralino coannihilation with chargino and next–to–lightest neutralino [47, 48].

5. Results

The allowed ranges of the SI cross section that result from our scans are illustrated in Fig. 1 for $\tan\beta = 35$, $A_t = A_b = 1\,\mathrm{TeV}$ and $\mu > 0$. Since σ_p^{SI} generally grows with $\tan\beta$ due to an enhancement in the heavy scalar Higgs coupling to down–type quarks, the above choice of $\tan\beta$ is a reasonable compromise between low and very large values. In deriving the allowed ranges we have imposed the bounds from colliders and $b \to s\gamma$, and further required $0.1 < \Omega_\chi h^2 < 0.2$, as described earlier, but *not* yet the bound from $(g-2)_\mu$. We also mark with a thick solid line a *parameter–independent* lower bound on σ_p^{SI} which will be explained below.

We can see a big spread of σ_p^{SI} over some seven (four) orders of magnitude at small (large) m_χ. In fact, the upper values of σ_p^{SI} exceed the latest experimental limits, including the very recent result from Edelweiss [4] and the preliminary limit from the UKDMC Zeplin I detector [5] which is expected to reach the level of $1.4 \times 10^{-6}\,\mathrm{pb}$ very soon. Also shown is the CDMS bound and the so–called DAMA region. It is clear that today's

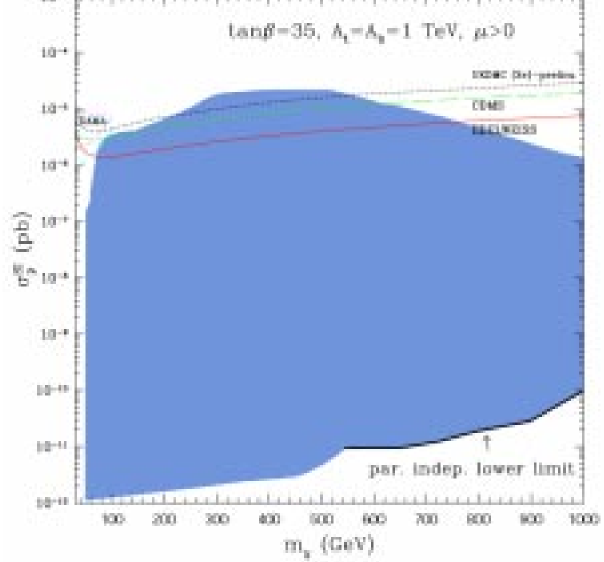

Figure 1: Ranges of σ_p^{SI} in the general MSSM *vs.* m_χ for $\tan\beta = 35$, $A_t = A_b = 1\,\mathrm{TeV}$ and $\mu > 0$, which are allowed by collider bounds, $b \to s\gamma$ and $0.1 < \Omega_\chi h^2 < 0.2$, but not from $(g-2)_\mu$. Also marked are some results of recent experimental WIMP searches. The thick black line indicates a *parameter–independent* lower bound at $m_\chi > 550\,\mathrm{GeV}$. No similar bound can be set for lower m_χ because of the neutralino–slepton coannihilation effect, as explained in the text.

experiments have already started probing the most favored ranges of σ_p^{SI} that come from SUSY predictions for neutralino cold dark matter.

In the four windows of Figs. 2 we show the effect of the most important constraints on the upper and lower limits on the allowed ranges of σ_p^{SI}. Firstly, in the upper left window we show the effect of relaxing the cosmological bound by allowing $\Omega_\chi h^2 < 0.1$. Obviously, larger ranges of σ_p^{SI} now become allowed since an enhancement in the neutralino pair–annihilation cross section often, by crossing symmetry, implies an increase in σ_p^{SI}. Note, however, that a combination of all the other constraints, most notably a lower limit on m_h from LEP, prevents σ_p^{SI} from rising by more than about an order of magnitude and only for not very large m_χ.

In the same window we also show an important effect, already pointed out in [21], of including neutralino coannihilation with sleptons (predominantly with the lighter stau) on allowing very low ranges of σ_p^{SI} at smaller m_χ. At lowest $m_\chi \lesssim 120\,\mathrm{GeV}$ the relic abundance can be reduced to the favored range by choosing in the scan light enough sleptons, even without coannihilation. By simultaneously choosing large enough heavy Higgs and squark masses, one can reduce σ_p^{SI} to very low values of a few $\times 10^{-12}\,\mathrm{pb}$. As m_χ increases, $\Omega_\chi h^2$ would normally increase as well, and become too large, but it is there that neutralino–slepton coannihilation kicks in. Since σ_p^{SI} is independent of the slepton masses, by carefully scanning the parameter space, one can always find $m_{\widetilde{l}}$ not much above m_χ, in which case $\Omega_\chi h^2$ can be sufficiently reduced again to fall into the favored range. The effect is very strong for smaller m_χ, thus explaining a sharp rise of the left side of the dark–red region allowed by neutralino–slepton coannihilation, but, as the process becomes increasingly inefficient at larger m_χ [27, 46], it gradually fades away.

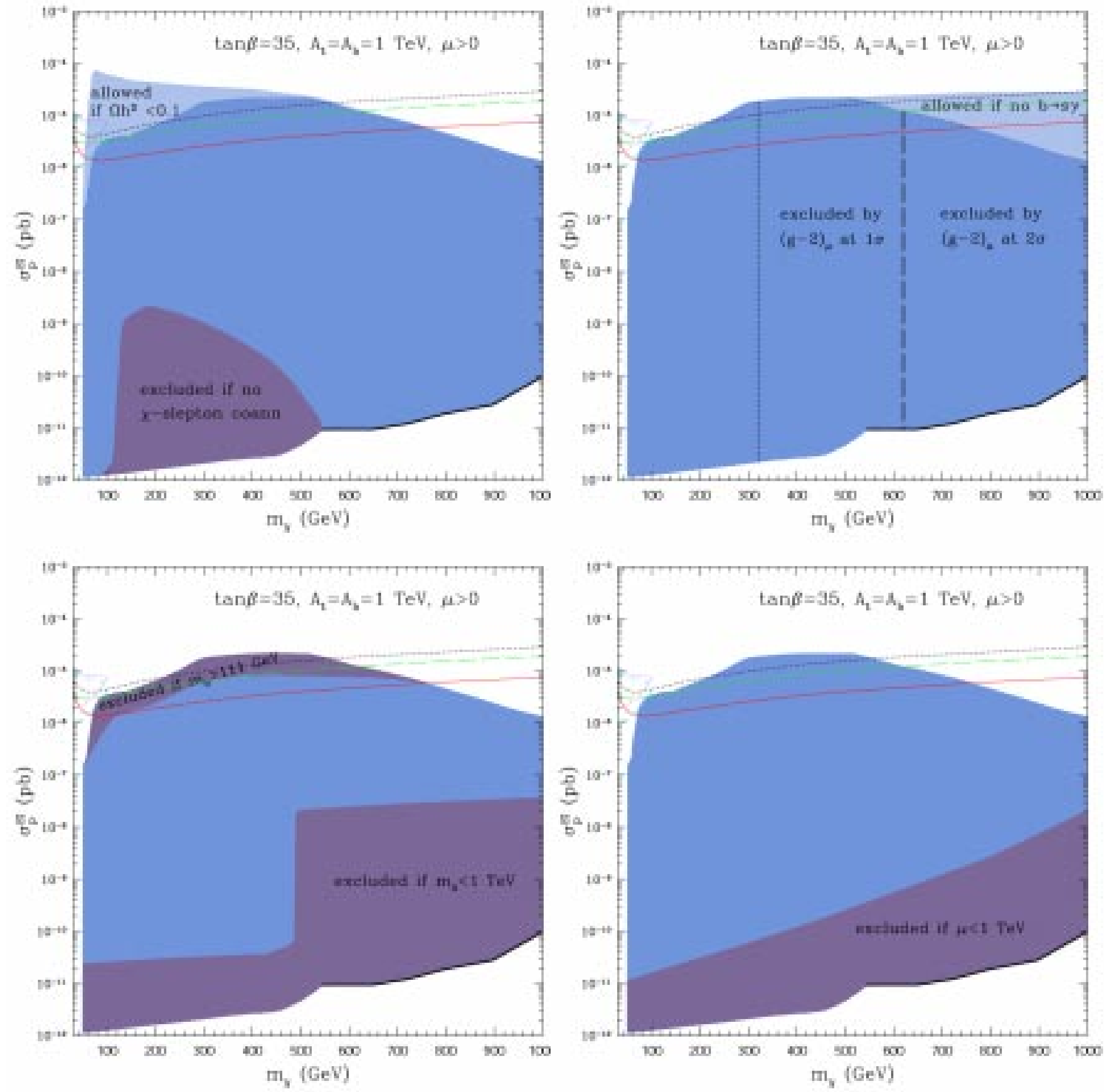

Figure 2: Sensitivity of upper and lower limits on σ_p^{SI} in Fig. 1 to various assumptions and constraints. Upper left window: the light blue region would be allowed if $\Omega_\chi h^2 < 0.1$. The dark–red region would be excluded if one neglected the effect of neutralino–slepton coannihilation. Upper right window: the light blue region would be allowed if one lifted the constraint from $b \to s\gamma$. The regions to the right of the vertical dotted (dashed) lines are excluded by imposing current 1σ (2σ) CL bound from $(g-2)_\mu$. Lower left window: the upper dark–red region would be excluded by assuming $m_h > 111\,\mathrm{GeV}$ for all m_A (*i.e.*, by neglecting a window of lighter m_h which is still allowed for $m_A < 120\,\mathrm{GeV}$). Also shown in this window is the effect of restricting $m_A < 1\,\mathrm{TeV}$. Lower right window: the same as for m_A but for μ.

In the upper right window of Fig. 2 we present the effect of imposing the constraint from $(g-2)_\mu$. We can see that, for this case, $m_\chi \lesssim 320\,\mathrm{GeV}$ (1σ CL) and $m_\chi \lesssim 620\,\mathrm{GeV}$ (2σ CL). This upper limit comes from the fact that, as m_χ increases, the SUSY contribution from the $\chi - \widetilde{\mu}$ and $\chi^- - \widetilde{\nu}_\mu$ loops become suppressed and at some point becomes too small to explain

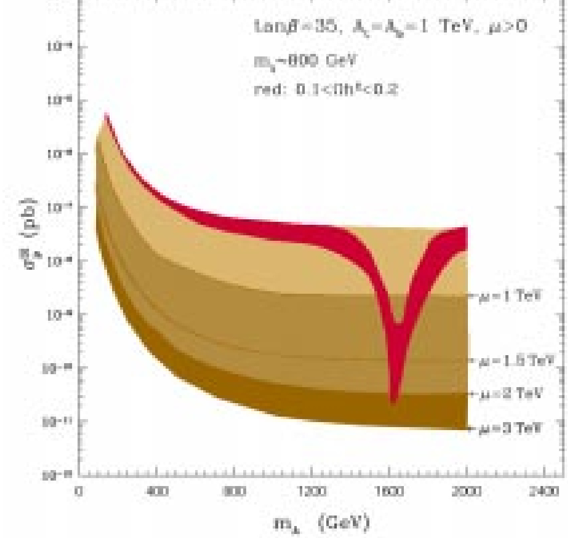

Figure 3: Sensitivity of σ_p^{SI} to m_A and μ for the case of Fig. 1. We concentrate on the region of parameter space where $m_\chi \sim 800\,\mathrm{GeV}$. The whole marked region is consistent with all the constraints from colliders and $b \to s\gamma$. By imposing further the constraint $0.1 < \Omega_\chi h^2 < 0.2$ one selects only the red region. Near the resonance $m_A \simeq 2m_\chi$, significantly smaller values of σ_p^{SI} become allowed by $0.1 < \Omega_\chi h^2 < 0.2$, (mostly in the increasingly pure bino region), but become eventually limited from below, independently of increasing the maximum allowed value of μ.

the apparent discrepancy between the SM and the experimental measurement [49, 50]. On the other hand, the allowed ranges of σ_p^{SI} are not really affected.

In the same window we also show the effect of relaxing the constraints from $b \to s\gamma$. We can see that if it were not imposed, the upper limit on σ_p^{SI} would significantly increase at large m_χ. In this region, the mass of the pseudoscalar Higgs, and therefore also the heavier scalar, is rather small, thus giving larger σ_p^{SI}. However, the mass of the charged Higgs is then also on the lower side, and a cancellation between a charged Higgs–top quark loop and chargino–stop loop contribution is not sufficient to reduce $\mathrm{BR}(B \to X_s\gamma)$ to agree with the experimental limit.

However, we remind the reader that a slight relaxation of the underlying assumption of minimal flavor violation in the squark sector often leads to a significant weakening of the bound from $b \to s\gamma$ to the point of even allowing $\mu < 0$ [38]. We would therefore be cautious in applying the $b \to s\gamma$ constraint too rigidly.

In the lower left window of Fig. 2 we present the sensitivity of the *upper* limit on σ_p^{SI} to the lower limit on the light Higgs mass. We can see that it would sizably decrease if we neglected the region of small $90\,\mathrm{GeV} \lesssim m_h \lesssim 111\,\mathrm{GeV}$ which is still allowed at $m_A \lesssim 120\,\mathrm{GeV}$, and instead required $m_h > 111\,\mathrm{GeV}$ for all m_A. Note also that the new experimental limits are for the most part inconsistent with the possibility of the light Higgs scalar.

In the same window and in the lower right window we explore the existence of the *lower* limit on σ_p^{SI} and its dependence on the assumed *upper* limit on m_A and μ, respectively. As we can see, the lowest values of σ_p^{SI} are often to a large extent determined by a somewhat subjective restrictions from above on these parameters. As one allows either μ or m_A above

1 TeV the lower limit on σ_p^{SI} relaxes considerably.

However, we argue that, at large enough m_χ, it is possible to set a *parameter–independent* lower bound on σ_p^{SI} (marked with a thick solid line in Fig. 2), independently of how large μ and other SUSY parameters are taken. Its origin is displayed in Fig. 3 where we plot σ_p^{SI} as a function of the pseudoscalar Higgs m_A for $m_\chi \simeq 800\,\mathrm{GeV}$ and scan over all the other parameters. As m_A and other parameters are varied, large ranges of σ_p^{SI} are allowed by collider and indirect constraints, depending on the maximum allowed value of μ. For each fixed μ, σ_p^{SI} decreases proportional to the fourth power of $m_H \simeq m_A$ because of the (typically dominant) t–channel exchange of the heavy Higgs, as the marked cases of μ demonstrate. As μ increases, at fixed m_χ one moves deeper into the gaugino region and typically finds large $\Omega_\chi h^2 > 0.2$. By imposing $0.1 < \Omega_\chi h^2 < 0.2$ one selects only a narrow red (dark) range with a large bino component. The bino purity ($p_{\widetilde{B}} = N_{11}^2$) increases with increasing μ, which normally quickly gives too large $\Omega_\chi h^2$. However, for each m_χ one can choose $m_A \simeq 2m_\chi$ (roughly $m_A = 1600\,\mathrm{GeV}$ in Fig. 3) in which case $\Omega_\chi h^2$ is reduced to an allowed level by a wide resonance due to A–exchange. This leads to allowing a much reduced σ_p^{SI}, while still being consistent with $0.1 < \Omega_\chi h^2 < 0.2$. However, because of the finite width of the A–resonance, at large enough μ one reaches the *lowest* value of σ_p^{SI} (in this case $2 \times 10^{-11}\,\mathrm{pb}$) which is *parameter–independent*. Away from the resonance (for example, if one imposed $m_A < 1\,\mathrm{TeV}$), one would obtain the lower bound $\sigma_p^{SI} \gtrsim 3 \times 10^{-8}\,\mathrm{pb}$, basically independently of whether $\mu < 1\,\mathrm{TeV}$ is imposed or not. In fact, one can see this effect in the lower left window of Fig. 2 where imposing $m_A < 1\,\mathrm{TeV}$ causes the lower limit on σ_p^{SI} to suddenly jump up at around $m_\chi \simeq m_A/2 \simeq 500\,\mathrm{GeV}$.

The above parameter–independent lower limit arises in the gaugino case. For higgsino–like LSP ($M_2 \gg \mu \simeq m_\chi$), $\Omega_\chi h^2$ remains typically very small $\Omega_\chi h^2 \ll 0.1$ and also σ_p^{SI} comes out some two orders of magnitude larger. In the mixed case σ_p^{SI} is even larger. On the other hand, at lower m_χ, coannihilation with sleptons prevents one from deriving a firm lower limit on σ_p^{SI}. Indeed, by suitably choosing the slepton mass not too much above m_χ, we can always reduce $\Omega_\chi h^2$ below 0.2. Thus, it is possible to set firm lower limits on σ_p^{SI} but only for large enough m_χ and even though they correspond to extremely large values of μ and accordingly involve much fine–tunning.

The dependence of σ_p^{SI} on A_t and A_b is rather weak. As the tri–linear terms deviate from zero, the mass of the lightest Higgs generally increases due to somewhat larger mass splittings among the stops and sbottoms. As a result, the normally subdominant contribution to σ_p^{SI} from the t–channel h–exchange is slightly reduced.

A far more important effect is that the number of SUSY configurations satisfying all experimental constraints, especially $b \to s\gamma$, decreases significantly. This is because the cancellation between charged Higgs loop and chargino loop contribution to the $b \to s\gamma$ decay rate becomes more inefficient as A_t decreases. For example, for $A_t = -1\,\mathrm{TeV}$ only a handful of points remain allowed. Generally, we have concluded that the regions allowed by lower values of A_t and A_b fall into the regions allowed by the choice $A_t = 1\,\mathrm{TeV}$.

In the left and right window of Fig. 4 we present the cases of $\tan\beta = 10$ and 50, respectively. Note that, for small $\tan\beta = 10$ the largest allowed values of σ_p^{SI} are roughly an order of magnitude smaller than at $\tan\beta = 50$ because of the $\tan\beta$–dependence of

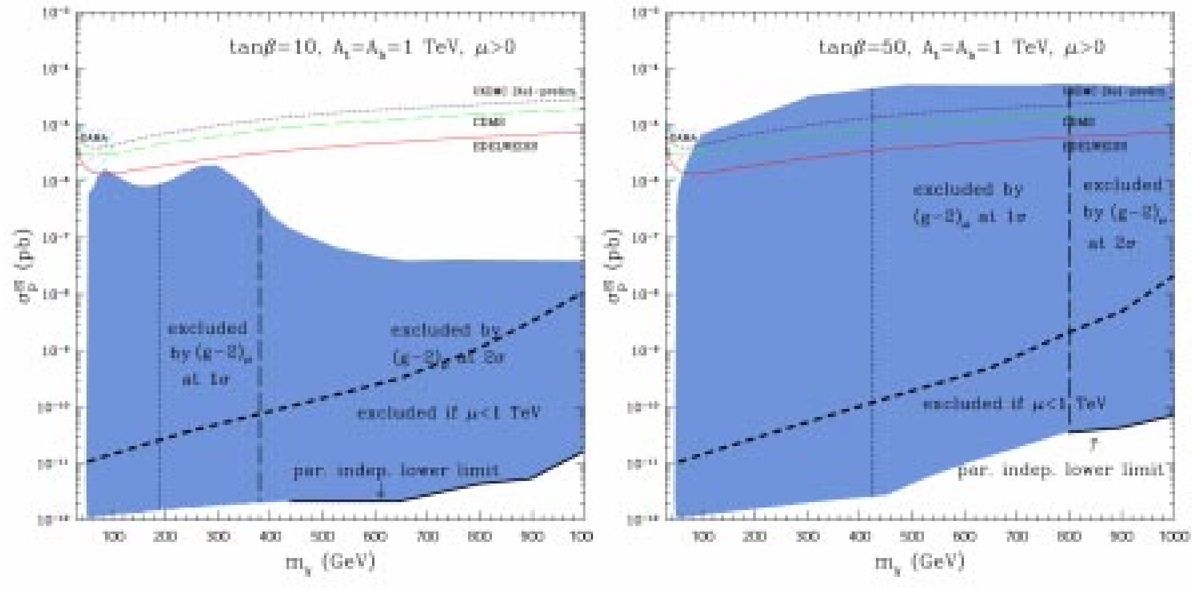

Figure 4: The same as in Fig. 1 but for $\tan\beta = 10$ (left window) and 50 (right window). Also marked is the effect of imposing $\mu < 1\,\mathrm{TeV}$.

the heavy scalar coupling to down–type quarks, as mentioned earlier. Notice a significant decrease in the upper ranges of σ_p^{SI} at large m_χ for $\tan\beta = 10$, which is caused by the $b \to s\gamma$ constraint. It is clear that the constraint is more severe in the case $\tan\beta = 10$ rather than at larger $\tan\beta$. This may sound somewhat counter–intuitive since, for example, in the Constrained MSSM, the constraint from $b \to s\gamma$ on the (CMSSM) parameter space becomes more pronounced at larger $\tan\beta$. This is because, in the CMSSM, the psedoscalar Higgs mass is typically large and a cancellation with chargino loop contribution would give too small $\mathrm{BR}(B \to X_s\gamma)$, below the lower experimental limit on $\mathrm{BR}(B \to X_s\gamma)$. In contrast, in the general MSSM case, we can choose small m_A values because m_A is a free parameter. This small m_A implies large positive charged Higgs loop contribution to $\mathrm{BR}(B \to X_s\gamma)$. In the large m_χ region, the chargino loop cannot cancel the charged Higgs contribution anymore and one exceeds the upper experimental limit on $\mathrm{BR}(B \to X_s\gamma)$. Because the chargino loop contribution is proportional to $\tan\beta$, the constraint from $b \to s\gamma$ is more severe for smaller $\tan\beta$. A similar effect can be observed from the $(g-2)_\mu$ constraint. For example, imposing a 1σ bound implies $m_\chi \lesssim 180\,\mathrm{GeV}$ for $\tan\beta = 10$, while for $\tan\beta = 50$ the bound remains at roughly $m_\chi \lesssim 420\,\mathrm{GeV}$, as denoted in the two windows of Fig. 4. Also marked is the effect of imposing $\mu < 1\,\mathrm{TeV}$. While not being a firm constraint, it does, in our opinion, indicate the region which may be considered as somewhat less fine–tuned.

At larger m_χ we can again set up the parameter–independent lower limit on σ_p^{SI}, in analogy with the case $\tan\beta = 35$. In order to do so, we had to explore extremely large ranges of μ up to some $4\,\mathrm{TeV}$ at smaller $\tan\beta$ in order to saturate the bound $\Omega_\chi h^2 < 0.2$. On the other hand, the lower limit on σ_p^{SI} at smaller m_χ remains basically independent of $\tan\beta$ since $\Omega_\chi h^2$ there is determined mostly by the coannihilation with sleptons, as discussed in detail in the case $\tan\beta = 35$. It arises from restricting μ below the value for which the parameter–independent lower limit arises at larger m_χ. Notice that the thick

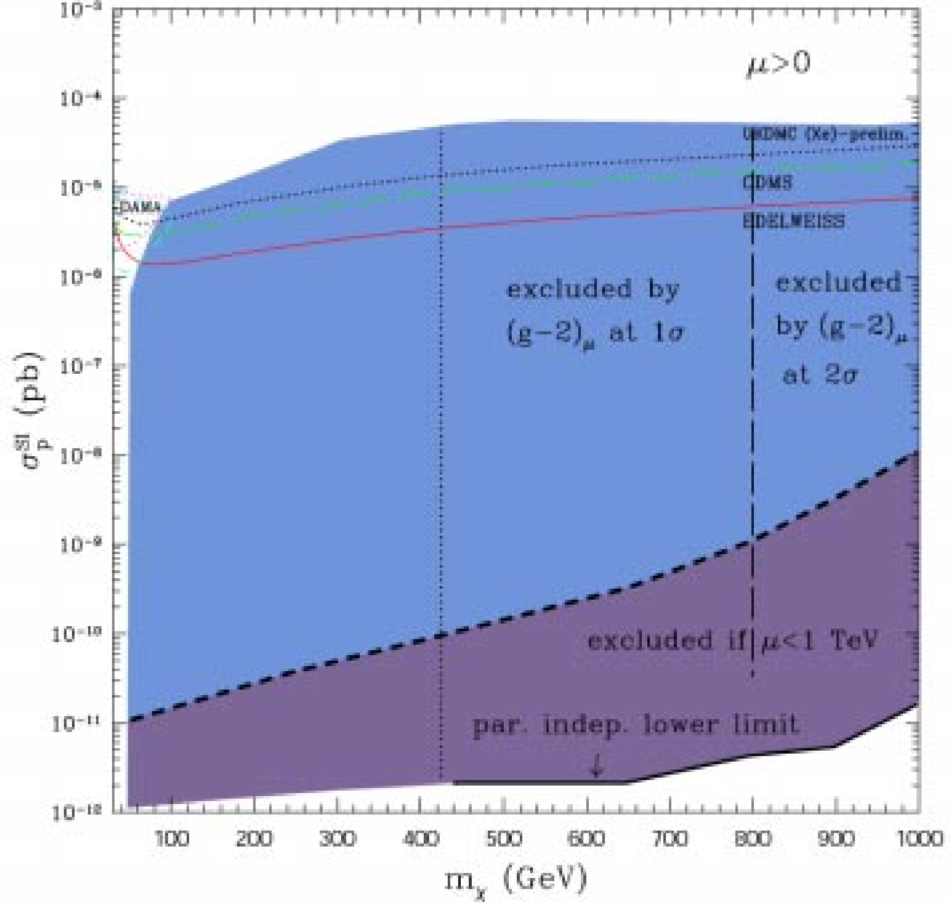

Figure 5: Ranges of σ_p^{SI} in the general MSSM *vs.* m_χ for $\mu > 0$, which are allowed by collider bounds, $b \to s\gamma$ and $0.1 < \Omega_\chi h^2 < 0.2$. Also marked are some results of recent experimental WIMP searches. The thick black line indicates a *parameter–independent* lower bound. The region below the dashed line is excluded if one imposes the constraint $\mu < 1\,\mathrm{TeV}$. The ranges of m_χ to the vertical lines are excluded at 1σ and 2σ CL by the current discrepancy between the experimental value of $(g-2)_\mu$ and the Standard Model prediction.

line extends to lower m_χ at smaller $\tan\beta$ because the efficiency of the neutralino–slepton coannihilation increases with $\tan\beta$.

Finally, in Fig. 5 we summarize the results for the full scan conducted so far for $\tan\beta = 5, 10, 35, 50$ and for $\mu > 0$. We repeat that, in determining the allowed (blue) region we applied the constraints from collider searches, $b \to s\gamma$ and $0.1 < \Omega_\chi h^2 < 0.2$. The lower limit on σ_p^{SI} mostly comes from low $\tan\beta$ and, where possible, we mark with the solid line the parameter–independent lower limit. The effect of restricting $\mu < 1\,\mathrm{TeV}$ is marked with a dashed line. Also marked is the impact of the new measurement of $(g-2)_\mu$ on the mass of the neutralino. If confirmed, this will imply rather stringent upper limits on m_χ

$$m_\chi \lesssim 420\,\mathrm{GeV} \quad (1\sigma\ CL) \tag{5.1}$$

$$m_\chi \lesssim 800\,\mathrm{GeV} \quad (2\sigma\ CL). \tag{5.2}$$

The vast ranges of σ_p^{SI} predicted in the framework of the general MSSM may be somewhat discouraging to DM WIMP hunters. It is worth noting, however, that it is the region of smaller m_χ, below a few hundred GeV, that is not only implied by the new result for $(g-2)_\mu$, but is also theoretically more favored as corresponding to less fine–tuning. Furthermore, ranges of very small $10^{-12}\,\mathrm{pb} \lesssim \sigma_p^{SI} \lesssim 10^{-8}\,\mathrm{pb}$ generally correspond either to very large (and therefore perhaps somewhat less natural) values of μ and/or m_A, or become allowed by selecting slepton masses on the light side, and in the χ–slepton coannihilation region, within some $20\,\mathrm{GeV}$ of m_χ, which again can be be considered as a finely–tuned case.

At the end, we comment again on the case of the Constrained MSSM. Because the model is much more restrictive, the ranges of σ_p^{SI} that one obtains in the parameter space allowed by all constraints, are very now much narrower [12, 13, 14, 15, 16]. They are also typically somewhat lower than the largest ones allowed in the general MSSM. For example, at $\tan\beta = 50$ we find $\sigma_p^{SI} \sim 10^{-7}\,\mathrm{pb}$ at $m_\chi = 100\,\mathrm{GeV}$ and $\sigma_p^{SI} \sim 7\times10^{-11}\,\mathrm{pb}$ at the largest (neglecting $(g-2)_\mu$) allowed value of $m_\chi = 800\,\mathrm{GeV}$. On the other hand, because the model is defined at the grand–unified, and not electroweak, scale, in the case of large $\tan\beta \gtrsim 50$ and/or large scalar masses, theoretical uncertainties involved in the running of parameters are substantial and have much impact on the resulting ranges of both m_χ and σ_p^{SI} [15]. We will explore the case of the Constrained MSSM in an oncoming publication.

6. Conclusions

We have delineated the ranges of the SI cross section σ_p^{SI} in the general MSSM, which are consistent with all the current experimental bounds and for which one finds the expected amount of dark matter. We have further discussed the dependence of our results on the underlying theoretical assumptions. While the ranges which we have obtained span over more than six orders of magnitude, we find it encouraging that the experimental sensitivity that has recently been reached, now allows one to explore our theoretical predictions for the MSSM. As we have argued above, smaller values of the WIMP mass and also larger values of σ_p^{SI} may be considered as more natural, which will hopefully be confirmed by a measuring a positive WIMP detection signal in the near future.

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